Shock magnetoplasma waves in metals

1995 J. Phys.: Condens. Matter 7 7549

(http://iopscience.iop.org/0953-8984/7/38/013)

J. Phys.: Condens. Matter 7 (1995) 7549–7559. Printed in the UK

Shock magnetoplasma waves in metals

L M Fisher†, N M Makarov‡, V E Vekslerchik‡ and V A Yampol'skii‡

† All-Russian Electrical Engineering Institute, Krasnokazarmennaya Street 12, Moscow 111250, Russia

‡ Institute for Radiophysics and Electronics, Ukrainian Academy of Sciences, Proscura Street 12, Kharkov 310085, Ukraine

Received 1 June 1995

Abstract. Propagation of strong magnetoplasma waves is studied with consideration of effects of the temporal dispersion. It is shown that the temporal dispersion in the nonlinear regime leads to overturning of the wavefront and generation of the shock waves. An analysis of the evolution of shock waves is carried out in both nondissipative and weakly dissipative cases.

1. Introduction

The present paper is devoted to the propagation of magnetoplasma waves in metals. These waves that were predicted by Kaner and Skobov [1] and first observed by Williams [2] have been studied rather well both theoretically and experimentally [3] in the past. However, it should be noted that the theoretical consideration has been restricted mostly to the case of small amplitudes, i.e., it has been carried out within the framework of the linear approximation. That is due to the fact that for a long time the electron theory of metals was developed exclusively as a linear science. It was presumed that nonlinear effects could hardly occur in metals because it is impossible to make the electron system depart considerably from its equilibrium state. That is why the traditional mechanisms of nonlinearity, say ones originating from overheating of electrons, do not work in metals. Only recently has it become clear that there is a mechanism of nonlinearity specific for metals working even under a weak deviation from the equilibrium. High conductivity, which suppresses the action of the already known mechanisms of nonlinearity, is a cause of a new one, specific to pure metals at low temperatures. This mechanism is a manifestation of the self-action of the field: a magnetic field of a wave affects the electron motion that causes the electric current that, in its turn, determines the field structure. This, the so-called magnetodynamic, mechanism of nonlinearity has been studied in a number of papers (see, e.g., the review [4] and references therein).

The magnetodynamic mechanism of nonlinearity causes a wide range of observable effects, such as a deviation of the current–voltage characteristics for thin metal samples from Ohm's law towards a decrease of the resistance [5, 6, 7], an appearance of the negative differential resistance [8], a pinch effect [9, 10, 11] and a generation of voltage auto-oscillations in the regime of a designated current [11]. In the radiowave range one should mention first a phenomenon of the 'current states' [12, 13, 14, 15, 16, 17], an appearance of dissipative structures of the electromagnetic field [18] and auto-oscillations [19, 20], a nonlinear attenuation [21, 22] and a nonlinear renormalization of a spectrum of

0953-8984/95/387549+11$19.50

electromagnetic waves [23]. Note that the above-mentioned effects are either static or low-frequency ones, and the aim of the present paper is to extend a study of the magnetodynamic mechanism into the region of higher frequencies where effects of the spatial and temporal dispersions of the electromagnetic field play a principal role.

It is well known that nonlinear processes can lead to an appearance of new field structures, having no analogues in the linear case, such as shock waves, solitons, kinks etc. The field distribution in media is determined, as a rule, by a joint action of several physical processes. Therefore, it is desirable to know what structures are typical for different mechanisms of nonlinearity (acting separately) to be aware of the tendencies that take place (and compete) in real situations. So, for example, solitons, that are generated when dispersion and overturning contend against each other, are, in some sense, intermediate between dispersing wave packets, which are typical of linear media, and shock waves, caused by some types of nonlinearity.

The spatial and temporal dispersions are naturally interrelated phenomena, but in this paper we, maybe somewhat artificially, shall separate them and restrict ourselves only to the consideration of the temporal one. So, the aim of the present work is to study the penetration of a strong wave into a compensated metal taking into account effects of the temporal dispersion.

2. Statement of the problem. Main equations

We shall consider a semi-infinite metal sample (it occupies the half space $x > 0$) that is irradiated by a monochromatic electromagnetic wave of frequency ω and amplitude $\mathcal{H}$. There is a constant magnetic field $\boldsymbol{H}_0$ parallel to the sample surface and the magnetic field of the wave. At the sample boundary $x = 0$ the value of the total magnetic field $H(x, t)$ is

$$H(0, t) = H_0 + \mathcal{H}\cos(\omega t). \tag{2.1}$$

The magnetic field $H(x, t)$ is assumed to be nonzero everywhere, i.e.,

$$\mathcal{H} < H_0. \tag{2.2}$$

The coordinate system is chosen as depicted in figure 1.

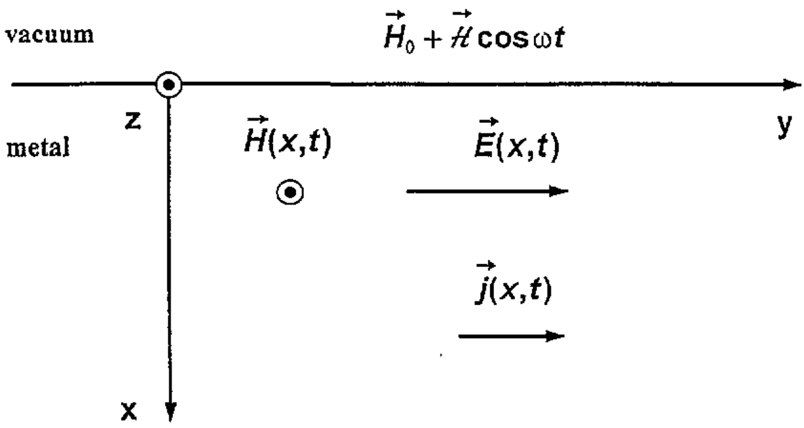

Figure 1. The geometry of the problem.

Let the external field H_0 be rather strong, i.e.,

$$\nu \ll \Omega \qquad \omega \ll \Omega \qquad \Omega = \min(\Omega_e, \Omega_h) \tag{2.3}$$

where ν is a relaxation frequency, $\Omega_{\rm e,h} = e_0 H_0/m_{\rm e,h}c$ are the electron and hole Larmor frequencies, e_0 is the elementary charge, $m_{\rm e,h}$ are the masses of electrons and holes, and c is the speed of light. In this case the temporal dispersion is weak and an expression for the current density in a compensated metal can be written as follows:

$$j_y = \frac{c^2}{4\pi V_{\rm A}^2}\frac{H_0^2}{H^2}\left(\dot{E} - \frac{\dot{H}}{H}E + \nu E\right). \tag{2.4}$$

Here a dot stands for differentiating with respect to time, $V_{\rm A}$ is the Alfven velocity,

$$V_{\rm A}^2 = \frac{H_0^2}{4\pi n\,(m_{\rm e} + m_{\rm h})} \tag{2.5}$$

n is the concentration of electrons (holes). A derivation of this formula is presented in the appendix. One can easily see that the expression obtained is essentially nonlinear with respect to H, which is typical for the case of strong magnetic fields. Formula (2.4) takes into account several current forming mechanisms, that is, first of all, a traditional dissipative mechanism. Actually, neglecting derivatives with respect to time in (2.4) one can obtain a classical expression for a dc current in a strong magnetic field, $\Omega \gg \nu$ (see, e.g., [25]). Besides, formula (2.4) describes the temporal dispersion effects. Partially these effects were taken into account by Kaner and Skobov. They used an expression for the current density that may be obtained from (2.4) at $H = H_0$. Such an approach corresponds to the situation when the ac amplitude $\mathcal{H}$ is much less than the dc external magnetic field H. One can say that the first summand in brackets in (2.4) describes a linear temporal dispersion, while the second one, proportional to $(\dot{H}/H)E$, is a nonlinear one. An expression for the current density similar to (2.4) has been used in [21, 22] where the authors studied the influence of weak temporal dispersion (both linear and nonlinear) on the propagation of linear waves.

For convenience we shall introduce dimensionless fields:

$$h = \frac{H}{H_0} \qquad e = \frac{c}{V_{\rm A}}\frac{E}{H_0}. \tag{2.6}$$

By means of these designations the Maxwell equations can be rewritten as

$$\dot{e} - \frac{\dot{h}}{h}e + V_{\rm A}h^2h' + \nu e = 0 \tag{2.7}$$

$$\dot{h} + V_{\rm A}e' = 0. \tag{2.8}$$

Hereafter a prime stands for differentiating with respect to the coordinate x. The boundary condition (2.1) in terms of h becomes

$$h(0,t) = h_{\rm in}(t) \tag{2.9}$$

where

$$h_{\rm in}(t) = 1 + a\cos(\omega t) \qquad a = \mathcal{H}/H_0. \tag{2.10}$$

To analyse nonlinear effects caused by the temporal dispersion we shall restrict ourselves to the situation of a weak dissipation,

$$\nu \ll \omega \tag{2.11}$$

and begin with a nondissipative limit.

3. Nondissipative case ($\nu = 0$)

The Maxwell equations at $\nu = 0$ read

$$\dot{e} - \frac{\dot{h}}{h}e + V_A h^2 h' = 0 \tag{3.1}$$

$$\dot{h} + V_A e' = 0. \tag{3.2}$$

Set (3.1), (3.2) is a quasilinear system of the hydrodynamical type. It possesses solutions with locally related e and h:

$$e = f(h). \tag{3.3}$$

Substituting (3.3) in (3.1), (3.2) one can obtain

$$\left(\frac{\mathrm{d}f}{\mathrm{d}h} - \frac{f}{h}\right)\dot{h} + V_A h^2 h' = 0 \tag{3.4}$$

$$\dot{h} + V_A \frac{\mathrm{d}f}{\mathrm{d}h} h' = 0. \tag{3.5}$$

Expressing, say, $\dot{h}$ from (3.5) and substituting it in (3.4) one can easily notice that system (3.4), (3.5) admits nontrivial (i.e., nonconstant) solutions only if the function f satisfies the following ordinary differential equation:

$$\frac{f}{h}\frac{\mathrm{d}f}{\mathrm{d}h} = \left(\frac{\mathrm{d}f}{\mathrm{d}h}\right)^2 - h^2 \tag{3.6}$$

which is nothing more than the compatibility condition for equations (3.4), (3.5) considered as an algebraic system for $\dot{h}$ and h'.

Solving (3.6) in a standard way, one can obtain

$$f(h) = h^2(\xi - \xi^{-1}) \tag{3.7}$$

where the function $\xi = \xi(h/h_0)$ is determined implicitly by

$$\left(\frac{h}{h_0}\right)^2 = \xi(2 - \xi^2)^{-3/2} \tag{3.8}$$

and h_0 is a constant that will be defined below.

In the case $\mathcal{H}/H_0 \ll 1$ (this case is the most interesting from the experimental point of view) formulae (3.7), (3.8) are simplified and the relation between e and h becomes explicit. It follows from (3.8) that the function ξ tends to unity as $\mathcal{H}/H_0 \to 0$. Thus, it can be considered as a measure of the nonlinearity of the problem. The expression for e as a function of h can now be written as

$$e = f(h) \simeq (h - h_0) + \tfrac{3}{4}(h - h_0)^2 \qquad \text{for } \mathcal{H}/H_0 \ll 1. \tag{3.9}$$

Using (3.7), one can reduce system (3.4), (3.5) to the well known (see, e.g., [24]) equation

$$\dot{h} + V(h)h' = 0 \tag{3.10}$$

with

$$V(h) = V_A h \xi(h). \tag{3.11}$$

Its solution satisfying boundary condition (2.10) may be presented implicitly as the solution of the following functional equation:

$$h = 1 + \frac{\mathcal{H}}{H_0} \cos\left\{\omega\left[t - \frac{x}{V(h)}\right]\right\}. \tag{3.12}$$

After the dependence $h(x, t)$ is found, the distribution of the electric field is given by (3.3).

An important fact following from the second Maxwell equation (3.2) is that the value of the electric field averaged with respect to the period of the incident wave is spatially homogeneous:

$$\langle e(x)\rangle = \frac{\omega}{2\pi}\oint \mathrm{d}t\, e(x, t) = \text{constant}. \tag{3.13}$$

Indeed, according to (3.2),

$$\frac{\partial}{\partial x}\langle e(x)\rangle = -V_{\mathrm{A}}^{-1}\frac{\omega}{2\pi}\oint \mathrm{d}t\,\frac{\partial}{\partial t}h(x, t) = 0 \tag{3.14}$$

since $h(x, t)$ is a periodic function, with the period $2\pi/\omega$.

According to the electrical neutrality of the metal, the constant in (3.13) should be taken as zero, i.e.,

$$\langle e(x)\rangle = 0. \tag{3.15}$$

From (3.15) with $x = 0$ one can obtain the condition $\langle e(0)\rangle = 0$ which should be used to determine the constant h_0. Thus, it follows from (2.10) that h_0 has to be a solution of the equation

$$\int_0^{2\pi/\omega} \mathrm{d}t\, h_{\mathrm{in}}^2(t)\left[\xi\left(\frac{h_{\mathrm{in}}(t)}{h_0}\right) - \xi^{-1}\left(\frac{h_{\mathrm{in}}(t)}{h_0}\right)\right] = 0 \tag{3.16}$$

where, remember, $h_{\mathrm{in}}(t) = 1 + a\cos(\omega t)$, $a = \mathcal{H}/H_0$. When a is small, h_0 can be expressed, up to the second order in a terms, as

$$h_0 \simeq 1 + \tfrac{3}{8}a^2 \qquad \text{for } \mathcal{H}/H_0 \ll 1. \tag{3.17}$$

Since $e(0, t) = f(1 + a\cos\omega t)$ (see (3.3)) and the function f is nonlinear, the electric field at the sample surface contains all harmonics. It can be easily shown, by expanding $e(0, t)$ in the Taylor series in a, that the nth harmonic is contributed by the terms $(a\cos\omega t)^{n+m}$ with $m = 0, 1, \ldots$. So, when $\mathcal{H} \ll H_0$ the nth harmonic of the electric field at the surface is of order $(\mathcal{H}/H_0)^n$. Using (3.9), one can easily obtain some first terms of the Fourier series for $e(0, t)$:

$$e(0, t) \simeq \left(a - \tfrac{3}{16}a^3\right)\cos\omega t + \tfrac{3}{8}a^2\cos 2\omega t \qquad \text{for } \mathcal{H}/H_0 \ll 1. \tag{3.18}$$

This immediately provides the expression for the surface impedance:

$$Z = \frac{4\pi}{c}\frac{E_\omega(0)}{\mathcal{H}} = \frac{4\pi V_{\mathrm{A}}}{c^2}\left[1 - \tfrac{3}{16}\left(\frac{\mathcal{H}}{H_0}\right)^2\right] \tag{3.19}$$

where $E_\omega(0)$ is the first harmonic of the electric field at the surface. Let us note here that the surface impedance does not contain the imaginary part and deviates from its linear value by the quantity of order $(\mathcal{H}/H_0)^2$.

Solutions (3.12), (3.9) admit passing to the linear limit ($\mathcal{H}/H_0 \to 0$). Rewriting (3.9) as $e = h - 1$ and putting $V(h) = V_{\mathrm{A}}$ in (3.12) one can obtain the expressions for the well known magnetoplasma wave.

Thus, equations (3.12), (3.9) describe the electromagnetic wave which is the nonlinear analogue of the magnetoplasma one. This fact is reflected, in particular, in expression

(3.19). Indeed, the surface impedance is real only in the case of the propagation of the undamped wave.

For small, but finite, values of $\mathcal{H}/H_0$ the main peculiarity of the obtained solutions is steepening and overturning of the wavefront, i.e., generation of the shock waves [24]. The distribution of the magnetic field, at a given moment of time, described by solution (3.12) is depicted in figure 2.

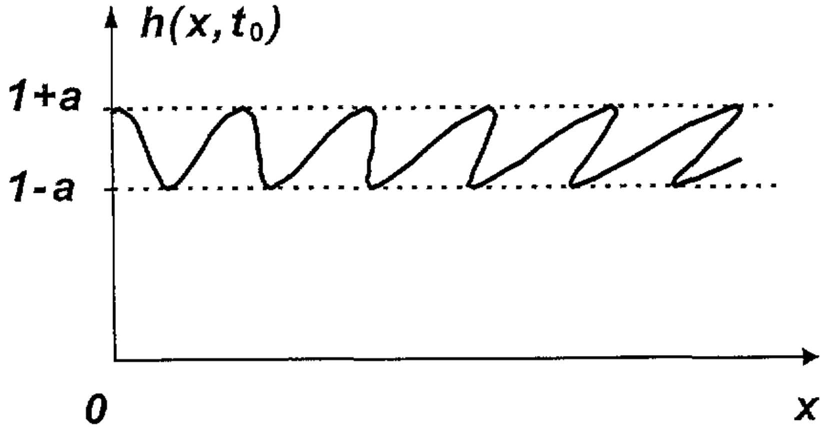

Figure 2. Distribution of the dimensionless magnetic field in the nondissipative case ($\nu = 0$) for $t = 0$, $a = 0.1$.

The points $x = x_0$ where the wavefront overturns are determined by the condition

$$h'(x_0, t) = \pm\infty. \tag{3.20}$$

It is easy to show that the overturning point can never be located at the surface of the metal ($x_0(t) \neq 0$). While propagating from the sample boundary, the wave oscillates many times with the scale $\lambda_A = V_A/\omega$, before the overturning takes place,

$$x_0(t) \approx \frac{V_A}{\omega}\frac{H_0}{\mathcal{H}} = \lambda_A \frac{H_0}{\mathcal{H}} \gg \lambda_A. \tag{3.21}$$

So, the considered electromagnetic wave is a chain of the shock waves and has saw-toothed shape only at distances from the sample surface large in comparison with the wavelength λ_A.

4. Weak-dissipation case ($\nu \ll \omega$). Current rectifying effect

The main effect caused by the dissipation is the fact that all oscillating components of the field (both magnetic and electric) attenuate while propagating from the irradiated surface into the metal bulk. In so doing the electric field tends to zero, while the magnetic one tends to some constant. In the linear situation the limiting value of the magnetic field H is the value of the external dc field H_0, or, in other words, $h \to 1$ as $x \to \infty$. In the case considered joint action of the nonlinearity and dissipation causes an effect of current rectifying, which leads to an appearance of the induced magnetic moment, i.e.,

$$\lim_{x\to\infty} h(x, t) = h_\infty \neq 1. \tag{4.1}$$

The value of h_∞ should be determined from the Maxwell equations (2.7), (2.8). At present, we cannot solve them for arbitrary value of the parameter $a = \mathcal{H}/H_0$, but if we restrict ourselves to the case $\mathcal{H} \ll H_0$, the problem of evaluating h_∞ becomes rather easy.

Averaging equation (2.7) over the period of the incident wave and using the fact that $\langle e\rangle = 0$ (see the derivation of formula (3.15)) one can get

$$V_A\left\langle h^2h'\right\rangle - \left\langle \frac{\dot h}{h} e\right\rangle = 0. \tag{4.2}$$

This relation, after some straightforward but rather cumbersome transformations, leads to the following one:

$$\frac{\partial}{\partial x}\left\langle \frac{h^3}{3} + \frac{e^2}{2h}\right\rangle = \frac{1}{2V_A}\left\langle \frac{e^2}{h^4}\left(\dot e - \frac{\dot h}{h} e + \nu e\right)\right\rangle. \tag{4.3}$$

Since the electric field e as well as the parameter a are small, we can neglect the r.h.s. in equation (4.3). Then, after integrating over x from zero to infinity, taking into account that $\langle h^3\rangle \to h_\infty^3$, $\langle e\rangle \to 0$, and omitting the terms of the third order in a, one can get

$$h_\infty^3 = \left\langle h^3 + \tfrac{3}{2}e^2\right\rangle\big|_{x=0}. \tag{4.4}$$

According to (2.11), we can naturally assume that the dissipation does not change the value of the electric field near the surface in the main order in a. So, we can use result (3.18) obtained for the nondissipative case. Finally, the limiting value of the magnetic field can be written as

$$h_\infty \simeq 1 + \frac{3}{4}\left(\frac{\mathcal{H}}{H_0}\right)^2 \qquad \text{for } \mathcal{H}/H_0 \ll 1. \tag{4.5}$$

5. Weak shock waves

To investigate a joint action of the temporal dispersion and the dissipation we shall restrict ourselves to the situation when the amplitude of the incident wave is much less than the external dc magnetic field: $\mathcal{H} \ll H_0$. In this case we can neglect the term proportional to $\dot h$ in equation (2.7) and write the Maxwell equations as

$$\dot e + \nu e + V_A h^2 h' = 0 \tag{5.1}$$

$$\dot h + V_A e' = 0. \tag{5.2}$$

In what follows we shall solve system (5.1), (5.2) assuming the dissipation to be weak (2.11).

As was shown in section 4, the dissipation term in (5.1), however small it could be, leads to the qualitative transformation of the solution: instead of an undamped wave (see section 3) we have the field distribution with a constant value (4.5) at infinity. So, we cannot apply a simple perturbation theory starting from the solution of equations (5.1), (5.2) with $\nu = 0$. The arguments below are to ground our approach.

After excluding the electric field from (5.1) and (5.2) we can get the equation for h,

$$\ddot h - V_A^2\left(h^2h'\right)' + \nu \dot h = 0 \tag{5.3}$$

which can be rewritten as

$$\left(\frac{\partial}{\partial t} - V_A\frac{\partial}{\partial x}h\right)\left(\frac{\partial}{\partial t} + V_A h\frac{\partial}{\partial x}\right)h + \nu\dot h = 0. \tag{5.4}$$

Here we have factorized the second-order nonlinear wave operator, presenting it as a product of two first-order operators. Each of them is responsible for the waves running in one of two possible directions. This means that in the absence of dissipation (for $\nu = 0$) one may solve the first-order equation $\dot{h} + V_A hh' = 0$ (compare with (3.10)) instead of the second-order one.

It turns out that it is possible to continue the factorization procedure to take into account partially the dissipation as well. Indeed, one can straightforwardly check that equation (5.3) is equivalent to the following one:

$$\left[\frac{\partial}{\partial t} - V_A \frac{\partial}{\partial x} h + \frac{\nu}{6}\left(4 - \frac{h_\infty^{3/2}}{h^{3/2}}\right)\right]\left[\frac{\partial h}{\partial t} - V_A h \frac{\partial h}{\partial x} + \frac{\nu}{3}\left(h - \frac{h_\infty^{3/2}}{h^{1/2}}\right)\right] - \frac{\nu^2}{18} h \left(4 - \frac{h_\infty^{3/2}}{h^{3/2}}\right)\left(1 - \frac{h_\infty^{3/2}}{h^{3/2}}\right) = 0. \tag{5.5}$$

Inequality (2.11) enables us to neglect the term quadratic in ν in (5.5) and to reduce the problem to solving the simpler equation

$$\dot{h} + V_A hh' + \frac{\nu}{3} h \left(1 - \frac{h_\infty^{3/2}}{h^{3/2}}\right) = 0 \tag{5.6}$$

under the boundary conditions

$$h(0, t) = h_{in}(t) = 1 + a \cos \omega t \tag{5.7}$$

$$h(\infty, t) = h_\infty. \tag{5.8}$$

Just the solution of set (5.6)–(5.8) is suggested to be used as an acceptable approximation for the description of the field distribution. Using the method of characteristics one can get

$$h(x, t) = \left\{h_\infty^{3/2} + \left[h_{in}^{3/2}(t - \tau(x, t)) - h_\infty^{3/2}\right] \exp\left(-\frac{\nu}{2}\tau(x, t)\right)\right\}^{2/3} \tag{5.9}$$

where $\tau(x, t)$ is determined by the relation

$$\frac{x}{V_A} = \int_0^\tau \mathrm{d}\tau' \left\{h_\infty^{3/2} + \left[h_{in}^{3/2}(t - \tau') - h_\infty^{3/2}\right] \exp\left(-\frac{\nu}{2}\tau'\right)\right\}^{2/3}. \tag{5.10}$$

This distribution of the magnetic field is presented schematically in figure 3.

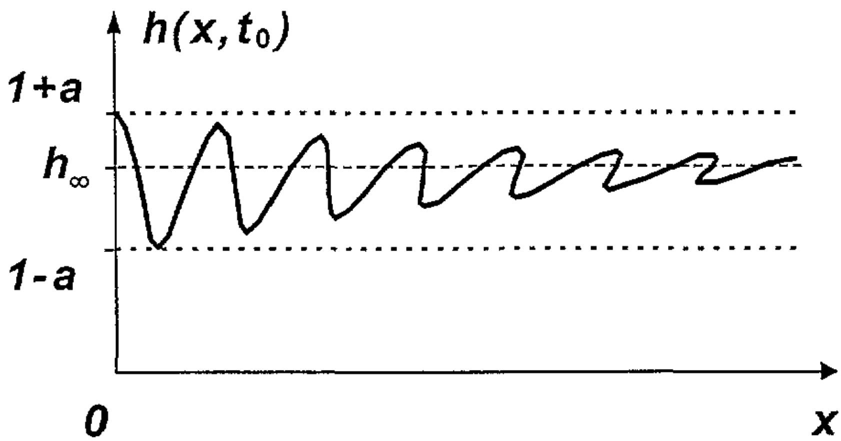

Figure 3. Distribution of the dimensionless magnetic field in the weakly dissipative case ($\nu \ll \omega$).

An analysis of the results obtained enables us to establish some features of the magnetic field distribution. So, it follows from (5.10) that τ, and consequently h, are periodic functions of t, i.e., the dissipation does not change the period of the temporal oscillations.

At large distances from the metal surface $\tau \approx x/V_\infty$, where

$$V_\infty = V_A h_\infty. \tag{5.11}$$

In that region the magnetic field distribution is given by

$$h(x,t) \approx h_\infty \left\{ 1 + \frac{2}{3} \left[\frac{h_{\rm in}^{3/2}(t - x/V_\infty)}{h_\infty^{3/2}} - 1 \right] \exp\left(-\frac{x}{L_\infty} \right) \right\} \tag{5.12}$$

with

$$L_\infty = 2\frac{\omega}{\nu} h_\infty \lambda_A \gg \lambda_A. \tag{5.13}$$

Thus, we have obtained the spatial attenuation law and the damping scale L_∞ of the wave.

It should be noted that a decrease of the wave amplitude is accompanied by the increase of the wave velocity in the metal bulk: $V_\infty > V_A$.

Besides, the dissipation hinders overturning of the wavefront. The formation of the shock waves does not occur if the value of ν is higher than some critical one, i.e. a threshold effect exists. One can obtain a qualitative criterion for the occurrence of discontinuity in the wave solution. The shock waves exist if the overturning scale x_0 (see (3.21)) is much higher than the dissipation length L_∞, or

$$\frac{\mathcal{H}}{H_0} \gg \frac{\nu}{\omega}. \tag{5.14}$$

A more careful analysis performed on the basis of the methods [24] gives a similar result.

6. Conclusion

We have considered the propagation of a strong magnetoplasma wave taking into account effects of the temporal dispersion. It has been shown that the magnetodynamic mechanism of nonlinearity leads to the overturning of the wavefront and generation of the shock waves. It should be noted that far from all questions related to the theory of the shock magnetoplasma waves have been discussed, that is, first of all, the problem mentioned in section 3 of the wavefront description. To solve this problem correctly it is necessary to include in our consideration the mechanisms that stabilize the overturning. The most important of them in our opinion is the spatial dispersion. Other questions are related to the geometry of the problem. We have considered a semi-infinite sample, that enabled us to obtain the shock wave 'in pure form'. It seems interesting to study an influence of the boundaries, all the more since in the nonlinear propagation regime the superposition principle does not hold, and rereflections of waves can lead to nontrivial results. These and some other questions are worthy of special consideration.

Acknowledgments

The research described in this publication was made possible in part by grant No N3F000 from the International Science Foundation and Russian Government and was supported by the Ukrainian Committee on Science and Technology (project No 2.3/19 'Metal') and the Russian Fundamental Science Foundation (project No 93-02-2039).

Appendix

The electrodynamics of metals is described by the Maxwell equations

$$\mathrm{curl}\, \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j} \qquad \mathrm{curl}\, \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \tag{A.1}$$

with the current density $\boldsymbol{j}$ given by

$$\boldsymbol{j} = -\frac{2e_0 m_e^3}{(2\pi\hbar)^3}\int f\boldsymbol{v}\,\mathrm{d}^3\boldsymbol{v}. \tag{A.2}$$

The distribution function f, depending on the position vector $\boldsymbol{r}$ and the velocity $\boldsymbol{v}$, should be found from the kinetic Boltzmann equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}} + \dot{\boldsymbol{v}}\frac{\partial f}{\partial \boldsymbol{v}} + \nu\,(f - f_{\mathrm{F}}) = 0 \tag{A.3}$$

where f_{F} is the Fermi distribution function depending on the energy $\varepsilon = m_{\mathrm{e}}v^2/2$ only. The kinetic equation is usually solved by the method of characteristics which may be defined by the motion equations for electrons,

$$m_{\mathrm{e}}\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -e_0\left\{\boldsymbol{E} + \frac{1}{c}\,[\boldsymbol{v}, \boldsymbol{H}]\right\}. \tag{A.4}$$

Generally speaking, system (A.1)–(A.4) should be solved self-consistently. However, this problem is unsolvable and usually it is simplified using various physical reasons. We shall assume $\boldsymbol{E}$ and $\boldsymbol{H}$ to be slowly varying functions of x and t and linearize the Boltzmann equation with respect to small electric field $\boldsymbol{E}$. This enables us to present the distribution function f as

$$f = f_{\mathrm{F}}(\varepsilon) - (\partial f_{\mathrm{F}}/\partial\varepsilon)\psi \tag{A.5}$$

and to obtain the following equation for the small addition ψ:

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi + \nu\psi = -e_0 E v_y. \tag{A.6}$$

Here we take into account the geometry of our problem and the fact that the Hall effect is absent in a compensated metal. Equation (A.6) can be easily solved:

$$\frac{\psi(t)}{e_0} = -\int_{-\infty}^{t}\mathrm{d}\tau\,\exp\{\nu(\tau - t)\}E(x(\tau), \tau)v_y(\tau). \tag{A.7}$$

The characteristics $x(\tau)$ and $\boldsymbol{v}(\tau)$ are determined by the equations

$$m_{\mathrm{e}}\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}\tau} = -\frac{e_0}{c}\,[\boldsymbol{v}, \boldsymbol{H}(x(\tau), \tau)]. \tag{A.8}$$

The magnetodynamic mechanism of nonlinearity is related to the magnetic component of the Lorentz force. That is why we neglect the term proportional to E but take into account the total magnetic field $\boldsymbol{H}(x, t)$ in (A.8). Using (A.8) one can take the integral in (A.7) by parts,

$$\frac{\psi(t)}{m_{\mathrm{e}}c} = \frac{E}{H}v_x - \int_{-\infty}^{t}\mathrm{d}\tau\,\exp\{\nu(\tau - t)\}\left\{\nu\frac{E}{H} + \frac{\mathrm{d}}{\mathrm{d}\tau}\frac{E}{H}\right\}v_x(\tau). \tag{A.9}$$

Since we consider the case of the weak dispersion (2.3) only, we can take the expression in braces outside the integral sign and use solutions of the motion equations at constant fields to evaluate the remaining integral. That leads to the following expression for ψ:

$$\frac{\psi}{m_{\mathrm{e}}c} = \frac{E}{H}v_x - \frac{cm_{\mathrm{e}}}{e_0 H}v_y\left\{\nu\frac{E}{H} + \frac{\partial}{\partial t}\frac{E}{H} + v_x\frac{\partial}{\partial x}\frac{E}{H}\right\}. \tag{A.10}$$

After substituting (A.10) into (A.2) and calculating the arising integrals, the expression for the electron current density takes the form

$$j_y = \frac{nm_e c^2}{H^2}\left\{\nu E + H\frac{\partial}{\partial t}\frac{E}{H}\right\}. \tag{A.11}$$

Taking into account an analogous contribution from the holes we obtain the expression (2.4) for the current density.

References

[1] Kaner E A and Skobov V G 1963 *Zh. Eksp. Teor. Fiz.* **45** 610

[2] Williams G A 1965 *Phys. Rev.* **139** 771

[3] Kaner E A and Skobov V G 1968 *Electromagnetic Waves in Metals in a Magnetic Field* (London: Taylor and Francis)

[4] Makarov N M and Yampol'skii V A 1991 *Fiz. Nizk. Temp.* **17** 547 (Engl. Transl. *Sov. J. Low Temp. Phys.* **17** 285)

[5] Kaner E A, Makarov N M, Snapiro I B and Yampol'skii V A 1984 *Zh. Eksp. Teor. Fiz.* **87** 2166 (Engl. Transl. *Sov. Phys.–JETP* **60** 1252)

[6] Kaner E A, Snapiro I B and Yampol'skii V A 1985 *Fiz. Nizk. Temp.* **11**(5) 477 (Engl. Transl. *Sov. J. Low Temp. Phys.* **11** 259)

[7] Voloshin I F, Kravchenko S V, Podlevskikh N A and Fisher L M 1985 *Zh. Eksp. Teor. Fiz.* **89** 233 (Engl. Transl. *Sov. Phys.–JETP* **62** 132)

[8] Kaner E A, Makarov N M, Snapiro I B and Yampol'skii V A 1985 *Zh. Eksp. Teor. Fiz.* **88** 1310 (Engl. Transl. *Sov. Phys.–JETP* **61** 776)

[9] Kaner E A, Leonov Yu G, Makarov N M and Yampol'skii V A 1987 *Zh. Eksp. Teor. Fiz.* **93** 2020 (Engl. Transl. *Sov. Phys.–JETP* **66** 1153)

[10] Makarov N M, Tkachev G B, Yampol'skii V A, Fisher L M and Voloshin I F 1995 *J. Phys.: Condens. Matter* **7** 625

[11] Zakharchenko S I, Kravchenko S V and Fisher L M 1986 *Zh. Eksp. Teor. Fiz.* **91** 660 (Engl. Transl. *Sov. Phys.–JETP* **64** 390)

[12] Babkin G I and Dolgopolov V T 1976 *Solid State Commun.* **18** 713

[13] Makarov N M and Yampol'skii V A 1983 *Zh. Eksp. Teor. Fiz.* **85** 614 (Engl. Transl. *Sov. Phys.–JETP* **58** 357)

[14] Dolgopolov V T and Chuprov P N 1983 *Solid State Commun.* **48** 165

[15] Makarov N M and Yampol'skii V A 1985 *Fiz. Nizk. Temp.* **11** 482 (Engl. Transl. *Sov. J. Low Temp. Phys.* **11** 262)

[16] Voloshin I F, Kravchenko S V and Fisher L M 1987 *Zh. Eksp. Teor. Fiz.* **92** 1050 (Engl. Transl. *Sov. Phys.–JETP* **65** 596)

[17] Makarov N M, Tkachev G B, Yampol'skii V A and Perez Rodriguez F 1993 *J. Phys.: Condens. Matter* **5** 7469

[18] Kaner E A, Makarov N M, Yurkevich I V and Yampol'skii V A 1987 *Zh. Eksp. Teor. Fiz.* **93** 274 (Engl. Transl. *Sov. Phys.–JETP* **66** 158)

[19] Babkin G I, Dolgopolov V T and Chuprov P N 1978 *Zh. Eksp. Teor. Fiz.* **75** 1801 (Engl. Transl. *Sov. Phys.–JETP* **48** 907)

[20] Voloshin I F, Kravchenko S V and Fisher L M 1986 *Dokl. Akad. Nauk SSSR* **287** 107 (Engl. Transl. *Sov. Phys.–Dokl.* **31** 237)

[21] Voloshin I F, Podlevskikh N A, Skobov V G, Fisher L M and Chernov A S 1986 *Pis. Zh. Eksp. Teor. Fiz.* **44** 136 (Engl. Transl. *JETP Lett.* **44** 173)

[22] Vugal'ter G A and Demikhovskii V Ya 1976 *Zh. Eksp. Teor. Fiz.* **70** 1419 (Engl. Transl. *Sov. Phys.–JETP* **43** 739)

[23] Burdov V A and Demikhovskii V Ya 1991 *Zh. Eksp. Teor. Fiz.* **100** 647 (Engl. Transl. *Sov. Phys.–JETP* **73** 359)

[24] Whitham G B 1974 *Linear and Nonlinear Waves* (New York: Wiley)

[25] Ziman J M 1972 *Principles of the Theory of Solids* (Cambridge: Cambridge University Press)